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REMARKS *on the* VELOCITY *with which* FLUIDS *iſſue from* APERTURES *in the* VESSELS *which contain them.* *By the Rev.* MATTHEW YOUNG, *D. D. S. F. T. C. D. & M. R. I. A.*

Read Jan. 20th, 1798.

WHEN water iſſues from a ſmall aperture in the bottom or ſide of a veſſel, which is kept conſtantly full, it has been ſuppoſed, that the force accelerating the loweſt plate of water, of indefinitely little altitude, immediately over the orifice, is the weight of the incumbent water only; and therefore, that after the motion of the plate has once commenced, the preſſure of the incumbent column will be diminiſhed, and of conſequence, the force accelerating the plate, during its deſcent through its own altitude, will not be conſtant.

BUT, in fact, it is not the preſſure of the incumbent water, which accelerates the loweſt plate; for every plate of water immediately incumbent over the hole, abſtracting from all lateral preſſure, begins to be accelerated equally at the ſame moment; and therefore the incumbent column, excluſive of any lateral preſſure, could produce no increaſe of velocity, in proportion to its increaſed height. The force which really accelerates the iſſuing plate, is the preſſure of the ambient water, which ſurrounds the cylinder immediately over the aperture; and this lateral preſſure being communicated to the upper ſurface of the plate, muſt be as much encreaſed by the velocity of the ſuperior deſcending plate, as it is diminiſhed by that of the inferior iſſuing plate, ſo as to remain conſtantly of the ſame magnitude.

On this principle it can be eaſily demonſtrated, that the velocity with which water ſpouts from an aperture in the bottom or ſide of a veſſel, is equal to that which a heavy body would acquire in falling through the height of the fluid above the orifice.

This demonſtration, however, as Mr. Atwood obſerves, is true only on hypotheſis that the water ſuffers no reſiſtance, but iſſues in a cylindrical or priſmatic form correſponding to the hole. But, in fact, the velocity of the water according to theory will be diminiſhed by the friction of the particles againſt the edges of the orifice; from their mutual attraction, by which the iſſuing particles are retarded by thoſe which are ſtill in the veſſel, and have not acquired the velocity of thoſe which precede them; but principally from the obliquity of their motions.

For, as Chev. Du Buat obſerves, when water iſſues from an orifice, the particles will flow from all ſides, towards the orifice, with an accelerated motion, and in all directions. If the orifice be horizontal, that filament of particles, which anſwers to the centre of the hole, will deſcend in a vertical line, and will ſuffer no other reſiſtance than that of the friction cauſed by the exceſs of its velocity above that of the collateral filaments, or by the retardation which ariſes from the attraction ſubſiſting between them. The other filaments, after they have deſcended vertically for ſome time, are compelled to turn from their vertical courſe, and to approach the orifice in different curves; and when they arrive at it, their directions become more or leſs horizontal, according as they paſs nearer to or farther from the edge of the orifice. The motion therefore is decompoſed according to two directions, the one horizontal, which is deſtroyed by the equal and contrary reſiſtance of the filaments which are diametrically oppoſite; the other vertical, in proportion to which the quantity of water diſcharged is to be eſtimated. Hence we ſee, that the vertical velocity of the filaments decreaſes from the centre of the orifice to its circumference; and that the total diſcharge is leſs, than if all the filaments had iſſued vertically, in the ſame manner with that which anſwers to the centre of the aperture. It alſo follows, that the filaments which are nearer to the centre, moving faſter than thoſe which are nearer to the edges, the vein of the fluid, after it has iſſued from the orifice, will form a cone whoſe baſe is the orifice; that is to ſay, that its diameter will diminiſh, at leaſt, to a certain diſtance, becauſe the exterior filaments are gradually drawn on, in conſequence of their mutual attraction, by the interior filaments whoſe velocity is greater; whence there follows a diminution in the diameter of the vein.

This manner of accounting for the contraction of the vein ſeems more reaſonable than that which is given by Newton; as there appears to be no adequate cauſe for the acceleration of the water, after it has been diſcharged from the orifice.

The diminution of the mean velocity of the water, cauſed ſolely by the obliquity of the motions of the iſſuing particles, excluſive of any other impediment, may be thus determined:

Let *m n* (fig. 1.) be the diameter of the aperture in the veſſel ABDC filled with water: in whatever direction the water iſſues, its velocity in that direction will, in all caſes, be the ſame, becauſe the preſſure of fluids is the ſame in all directions; thus, whether a fluid ſpouts perpendicularly upwards or downwards, horizontally or obliquely, the ſpace through which it is projected, in a given time, is the ſame. Now to determine this direction, ſince the horizontal and vertical preſſures are equal, the iſſuing particles will aſſume the intermediate direction, which will therefore form an angle of 45°. with the plane of the orifice: its vertical velocity therefore will be leſs than its direct or total velocity in the proportion of the diagonal of a ſquare to its ſide, or as 7 to 5 nearly; but the particles of the central filament iſſue with the full velocity, due to the entire height of the water; therefore the velocity of the central particles will be to the mean velocity, as 7 to the mean between 7 and 5, or as 7 to 6. This is the diminution, as has been ſaid, which takes place in conſequence ſolely of the obliquity of the motions with which the particles iſſue from the orifice: if the other cauſes of retardation be taken into the account, we may conclude, that the velocity ſhould be diminiſhed perhaps in the ratio of 8 or even 9 to 6; which accords very well with experiments. Thus Polenus makes the ratio of the diameters of the contracted vein and aperture, which is the ſame with that of the mean and greateſt velocity, to be as 5½ to 6½; Bernouilli 5 to 7; Chev. Du Buat 6 to 9. When the orifice is infinitely little, the cylinder of iſſuing water becomes a ſingle filament, which is therefore diſcharged without any obliquity, and there will be no diminution of velocity, except ſuch as ariſes from friction and the tenacity of the particles. If the aperture be encreaſed ſo as to become equal to the baſe of the veſſel, the column of water will then deſcend like a falling body, and therefore the velocity will be the ſame as before; but it will not acquire this velocity until the uppermoſt plate of water has been diſcharged. At the beginning of the motion, the firſt or loweſt plate will flow out with a velocity indefinitely little; the next plate with a greater velocity; and ſo on, until the upper plate ſhall have deſcended to the orifice which will then iſſue with the greateſt velocity. But if the veſſel be ſuppoſed to be kept conſtantly full, the velocity of the effluent water will encreaſe ſo as at length to become equal to that which a heavy body would acquire in falling from an infinite height.

Since the middle filament of particles is diſcharged with the full velocity due to the entire altitude of the fluid above the orifice, experiments made on the diſtance or height to which fluids ſpout, will be found to agree very well with theory, but it by no means follows, that all the filaments ſhould be diſcharged with the ſame velocity: the quantity of the fluid therefore diſcharged in a given time, may be leſs than that which would be diſcharged, if all the filaments were diſcharged with the velocity due to the entire altitude; becauſe this quantity depends on the mean velocity of all the filaments. Hence therefore it cannot be inferred from theſe experiments, compared with thoſe which relate to the height or diſtance to which the fluid ſpouts, that the velocity of the water in the orifice is leſs than that which is due to the entire altitude; and that it is accelerated immediately after it gets out of it: becauſe the diſtance to which the fluid ſpouts, depends on the central filament only, but the quantity diſcharged on the mean velocity of the whole.

To bring this queſtion to the teſt of experiment, if all the particles were equally accelerated at their diſcharge from the orifice, and immediately after they leave it, they ought all to be projected horizontally to the very ſame diſtance upon an horizontal plane; but on experiment I found, that when the fluid ſpouted through an orifice of ,08 of an inch diameter, and was kept conſtantly at the ſame height, the greateſt and leaſt diſtances at which it ſtruck the horizontal plane were nearly 15 and 12 inches; but theſe diſtances are proportional to the velocities with which they are diſcharged. It follows therefore, that all the particles are not projected with the ſame velocity. It is to be obſerved, that the particles which are diſcharged with the greateſt and leaſt velocities are few in compariſon of thoſe which are diſcharged with intermediate velocities, for while the entire ſhower extended from 15 to 12 inches on the horizontal plane, the denſer part was found to occupy only the ſpace between 14¼ and 12¾ inches; ſo that the limits of the velocities of the parts of the denſer ſhower were as 7 and 6,26; but the limits of the whole were 15 and 12, or as 7 and 5,6; and the limits by theory are as 7 and 5. But we may perceive, that when the fluid ſpouts horizontally, the particles which iſſue from the upper part of the aperture, and which therefore ought to move with the leaſt velocity, muſt encounter thoſe below them moving with a greater velocity, which will encreaſe the diſtance to which they are projected on an horizontal plane. Likewiſe, the particles which iſſue from the loweſt part of the orifice, and which ought to move with a leſs velocity, than that with which thoſe in the axis move, in the ratio of 5 to 7, will have their velocity encreaſed by their being at a greater depth. The limit therefore of the ratio of the diſtances to which the particles are projected on an horizontal plane, muſt be leſs than that which reſults from the theory of water iſſuing through an horizontal aperture. But it is obvious that the greater depth of the lower particles, when the orifice is vertical, cannot account for the entire difference of diſtance to which the particles are projected; for the depth of the orifice being 8,55 inches, and the diameter of the orifice ,08 of an inch, the velocities on account of the difference of depth would be only as $\sqrt{8,55}$ to $\sqrt{8,63}$, or as 14,6 to 15 nearly. Perhaps it might be ſaid, that this difference of diſtance was cauſed, not by the different velocities, but by the different directions in which the particles are diſcharged; ſo that thoſe which are projected in the axis of the vein, will ſtrike the horizontal plane at a greater diſtance than thoſe which are projected from the edges of the orifice with the ſame velocity, but in a different direction. But this cannot be the cauſe; for when the aperture is horizontal, the particles which iſſue from the oppoſite ſides *m*, *n* of the orifice (fig. 2.) meeting each other, deſtroy their convergence, and afterwards proceed in the direction of the axis of the vein, and therefore the vein will continue nearly of the ſame diameter: whereas, if the particles croſſed each other, with the ſame velocity, in different directions, they would deſcribe interſecting parabolas *n s*, *m t*, and the diameter of the vein would continually encreaſe. In order to determine whether this were the caſe, I cauſed the fluid to iſſue through an aperture in the bottom of the veſſel, and at the diſtance of 12 inches I found the diameter of the vein a little encreaſed, when the velocity of the efflux was conſiderable;

ſiderable; but not ſenſibly augmented, when the velocity was much diminiſhed. Since the dilatation of the vein in this caſe depends on the velocity with which the water iſſues from the aperture, it is to be inferred, that it is cauſed by the reſiſtance of the air, which producing a retardation of the preceding particles, thoſe which follow impinge againſt them, and the thickneſs of the vein is encreaſed; for the ſame reaſon as when the jette is made perpendicularly upwards, a broad head is formed in conſequence of the retardation of the uppermoſt particles. Now ſince it appears, that the dilatation of the vein which ariſes either from the different directions of the particles, or the reſiſtance which they undergo from the air, or both together, cannot account for the difference of diſtance to which the particles are projected on an horizontal plane, we muſt conclude that this difference is cauſed by the different velocities with which they eſcape from the orifice.

When a tube *mnrs* (fig. 3) is inſerted into the veſſel ABCD, it is found, that the velocity is increaſed nearly in the ſub-duplicate ratio of the length of the pipe, when the tubes are ſhort; and that it approaches nearer to that ſub-duplicate ratio, according as the length of the pipe is increaſed. To account for this increaſe of velocity has appeared a matter of ſome difficulty, ſince the water cannot iſſue at *rs* with a greater velocity than it enters at *mn*; and it does not appear how the velocity at *mn* can be encreaſed by inſerting a tube beneath it. In order to explain the cauſe of this effect, we are to conſider, that the whole force with which the plate *m n* is preſſed down, is the weight of a column of water equal to *e m n f*, together with the weight of a column of air of the ſame baſe, reaching to the top of the atmoſphere; and the whole force with which it is preſſed up, is the weight of an equal column of air, diminiſhed by the weight of a column of water equal to *m n r s*; therefore the actual force with which the plate *m n* is preſſed down, is the weight of a column of water equal to *e f r s*; the velocity therefore with which the plate *m n* will iſſue through the orifice *m n*, will be the ſame as through the orifice *r s* in the veſſel A*bc*D; that is, equal to the velocity which a heavy body would acquire in falling through the altitude *e r*; and all the plates of water in the tube *m n r s* will deſcend with the ſame velocity; for they cannot deſcend faſter, becauſe otherwiſe there would be a vacuum left in the tube, which is prevented by the upward preſſure of the atmoſphere. And the velocity of the effluent water will be the ſame, whatever be the preſſure of the atmoſphere, provided the weight of a column of air of the ſame baſe with *r s*, and whoſe height is equal to that of the atmoſphere, be either greater than or equal to the weight of the pillar of water *m n r s*. This might be proved experimentally by a veſſel of water with a pipe inſerted in the bottom, placed under an exhauſted receiver. But as the operation of exhauſtion is obſtructed more by the evaporation of water than of mercury, it will be better to uſe mercury in theſe experiments. Now if D be the defect of the gage from the ſtandard altitude, it will meaſure the preſſure of the air on the ſurface of the mercury in the veſſel; let A be the altitude of the mercury in the veſſel above the upper orifice of the pipe, and P the length of the pipe; then the whole force preſſing downwards the plate of mercury which is immediately in the upper orifice of the pipe, will be $= D + A$; and the whole force preſſing the ſame plate upwards will be $D—P$; and the difference between theſe forces will be the abſolute force preſſing the ſame plate of mercury downwards; while D is greater than P, this abſolute force will conſequently be equal to $A + P$; when $D = P$, $D—P$ vaniſhes, and the force preſſing the plate downwards is $= D + A = P + A$; hence therefore no variation in the time of the efflux will be perceived, while the altitude of the mercury in the gage is equal to or leſs than the difference between the length of the pipe and the ſtandard altitude. When D is leſs than P, the force upwards is alſo nothing; and therefore, as before, the whole force preſſing the plate downwards is $= D + A$; and A being given, it decreaſes according as D decreaſes; and when D vaniſhes, that is, when the receiver is abſolutely exhauſted, the force becomes equal to A, and the time of the efflux will be the ſame, as if the pipe had not been inſerted in the bottom of the veſſel. To try the truth of theſe things by experiment, I inſerted a tube 7,8 inches long in a cylindrical veſſel, and cloſing the orifice of the pipe, I filled the veſſel with mercury to the height of 6 inches; then placing the apparatus under the receiver of an air-pump, when the barometer was at 30 inches, and the gage at 28,5; the time of the efflux was 26 ſeconds; when the experiment was repeated preciſely in the ſame manner, but in the open air, the time of the efflux was only 19 ſeconds. Now as the gage ſtood at 28,5, the defect D was $30 - 28{,}5 = 1{,}5$, and the preſſure on the plate of mercury was $= 6 + 1\frac{1}{2} = 7\frac{1}{2}$; in the open air the preſſure was $= 6 + 7{,}8 = 13{,}8$; therefore the ratio of the velocity of the efflux in both caſes, which is the ſame with the reciprocal ratio of the times, was $\sqrt{7\frac{1}{2}}$ to $\sqrt{13{,}8}$, or as 2,73 to 3,7; but 2,73 is to 3,7 as 19 to 26 very nearly. No difference was obſerved in the times of the efflux, when in the open air and exhauſted receiver, unleſs the gage ſtood higher than 22½ inches; that is, unleſs the height of the mercury in the gage was greater than the difference between the length of the pipe and the ſtandard altitude. In another experiment, when the gage ſtood at 27,9, the height of the barometer was 29,9; the defect therefore was $= 2$, and the preſſure $= 8$. But $\sqrt{8} = 2{,}828$, and $\sqrt{13{,}8} = 3{,}7$ but $2{,}828 : 3{,}7 :: 19 : 24$, and by experiment the time of the efflux appeared to be 23 ſeconds. When the efflux is made in vacuo, it is obvious to obſerve, that the pipe is not filled during the efflux, as it is while the diſcharge is made in the open air.

Since the column of water in the pipe *mnrs* adds to the preſſure which the plate *mn* ſuſtains, by diminiſhing the upward preſſure of the air through the pipe, it appears that it produces this increaſe of preſſure in the plate *mn* alone, without producing any lateral preſſure in the water which is on a level with *mn*; for it is manifeſt, that if an aperture were made in *m*B or *n*C, the velocity of the water iſſuing through it would not be affected by the inſertion of the pipe; and conſequently that the plate *mn*, which is immediately in the orifice of the pipe, is the only one, on the ſame level, whoſe tendency downwards is increaſed by the inſertion of the pipe. Hence, the particles of water at the edge of the aperture, having their perpendicular preſſure encreaſed by the weight of the column *mnrs*, without any increaſe of their lateral preſſure, they will iſſue through the orifice *mn* more perpendicularly; the ſides alſo of the tube will obſtruct the converging motion of the particles, and conſequently, on both theſe accounts, the quantity of water diſcharged through a pipe thus inſerted, will exceed that diſcharged through a ſimple orifice, in a greater ratio than the ſub-duplicate of the height of the water. And according as the length of the pipe encreaſes, the ratio of the quantity of water actually diſcharged by experiment, to that which ſhould be diſcharged according to theory, will increaſe; becauſe the ratio of the perpendicular to the horizontal preſſure increaſes, in the ratio of the ſum of the depth of the veſſel and length of the pipe, to the depth of the veſſel. It follows therefore, that experiments made in this manner, will approach nearer to coincidence with theory, than when made with a ſimple orifice; except either when the tube is ſo long as that the friction of the fluid againſt the ſides of the tube ſhall produce a ſenſible effect, or it is ſo ſhort, as not to be ſufficient to give the particles a vertical direction All which agrees very well with the experiments made by the ingenious Mr. Vince, of which he has given us an account in the Phil. Tranſ. for the year 1795. Thus he tells us, that having inſerted a tube, a quarter of an inch in length, into a cylindrical veſſel 12 inches deep, he found that the velocity did not ſenſibly differ from that through the orifice; the cauſe of which he diſcovered to be this, that the ſtream did not fill the pipe, but that the fluid was contracted, as when it flowed through the ſimple orifice. When the pipe was half an inch long, inſerted into a veſſel of the ſame depth as before, the velocity of the water from the pipe and from the orifice, which ought by theory to have been as $\sqrt{12{,}5}$ to $\sqrt{12}$, or as 49 to 48, was by experiment found to be nearly in the proportion of 4 to 3. Now if the ratio of 49 to 48 be increaſed in the ratio of 7 to 6, (becauſe this is the ratio of the diminution of the velocity on account of the contraction of the vein, and this contraction either nearly or entirely vaniſhes in a pipe,) we ſhall have the ratio of 3,57 to 3. When the pipe was an inch long, the velocity from the pipe and from the orifice, which, according to theory, ought to have been as $\sqrt{13}$ to $\sqrt{12}$, or as 26 to 25, appeared by experiment, very nearly in the ratio of 4 to 3; now if the ratio of 26 to 25 be encreaſed in the ratio of 7 to 6, we ſhall have the ratio of 3,64 to 3. When he made uſe of longer pipes, the velocity of the effluent water by experiment approached nearer to that which ought to have been diſcharged according to theory; ſo that in long pipes, the difference between theory and experiment, he ſays, was not greater than what might be expected from the friction of the pipes, and other cauſes which may be ſuppoſed to retard the velocity. When he inſerted a pipe of the ſame diameter with the aperture, which terminated in a truncated cone fixed in the orifice, (fig. 4.) he expected, that the quantity of water diſcharged in a given time would have been diminiſhed, becauſe the water, iſſuing through the orifice *mn*, would have room to form the *vena contracta* in the enlarging cone; but he found, that the ſame quantity of water was diſcharged, as if the pipe had continued throughout of the ſame diameter with the orifice. The reaſon of this is manifeſt from what has been ſaid, for the preſſure of the air will not ſuffer the truncated cone to remain partly empty, as it would be if the *vena contracta* were formed; it will therefore continue full, and conſequently the water will paſs through it in the ſame manner as if the water in the cone, ſurrounding the pipe *mabn*, were congealed.

Mr. Vince likewiſe inſerted into the bottom of the veſſel a perpendicular pipe, in form of a truncated cone, the narrower part being fixed in the orifice; by which he found the efflux to be encreaſed more than if he had inſerted a cylindrical pipe of the ſame length, whoſe diameter was equal to that of the narroweſt part of the conical pipe. This effect may be explained on the ſame

principle

principle by which we accounted for the augmentation of the diameter of a vertical vein of water, through a ſimple orifice, when the velocity of the efflux is conſiderable. For when a perpendicular pipe is inſerted, the velocity of the diſcharge being conſiderably encreaſed, the reſiſtance from the air will be ſo likewiſe; and thus the diameter of the vein has a tendency to enlarge itſelf; now in the widening cone, the pipe admits of this augmentation, at the ſame time that it encreaſes the velocity; but the cylindrical pipe, though it equally encreaſes the velocity, yet it does not permit the vein to enlarge itſelf, and by thus confining it, the efflux is obſtructed, and the quantity diſcharged in a given time is diminiſhed. Accordingly, under the receiver of an air-pump, even in a moderate degree of exhauſtion, there is no difference perceived between the velocities with which a fluid is diſcharged through a conical or cylindrical pipe.

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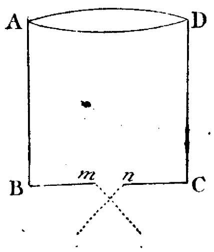

Fig. 1.

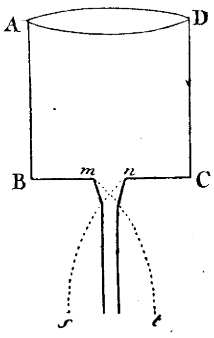

Fig. 2.

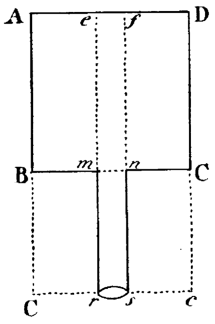

Fig. 3.